| Reg.No.: | | | | | | | | | | | | |
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VIVEKANANDHA COLLEGE OF ENGINEERING FOR WOMEN

[AUTONOMOUS INSTITUTION AFFILIATED TO ANNA UNIVERSITY, CHENNAI]

Elayampalayam – 637 205, Tiruchengode, Namakkal Dt., Tamil Nadu.

**Question Paper Code: 20011**

B.E. / B.Tech. DEGREE END-SEMESTER EXAMINATIONS – JAN. / FEB. 2025

Second Semester

Computer Science and Engineering

U23MA202 – COMPLEX ANALYSIS AND ORDINARY DIFFERENTIAL EQUATIONS

(Common to All)

(Regulation 2023)

Time : Three Hours    Maximum : 100 Marks

Answer ALL the questions

| Knowledge Levels (KL) | K1 – Remembering | K3 – Applying | K5 - Evaluating |
|---|---|---|---|
| | K2 – Understanding | K4 – Analyzing | K6 - Creating |

## PART – A

(10 x 2 = 20 Marks)

| Q.No. | Questions | Marks | KL | CO |
|---|---|---|---|---|
| 1. | Test whether the function $f(z)=\bar{z}$ is analytic or not. | 2 | K4 | CO1 |
| 2. | Find the fixed points of the bilinear transformation $w=\frac{z-1}{z+1}$. | 2 | K1 | CO1 |
| 3. | Find $\int_C \frac{z}{z-2}dz$, where C is the circle $\|z\| = 1$. | 2 | K1 | CO2 |
| 4. | State Cauchy's Residue theorem. | 2 | K1 | CO2 |
| 5. | Construct the work done in moving a particle in the force field given by $\vec{F} = 3x^2\vec{i} + (2xz-y)\vec{j} + z\vec{k}$ along the straight line from (0,0,0) to (2,1,3). | 2 | K3 | CO3 |
| 6. | State Stokes theorem. | 2 | K1 | CO3 |
| 7. | Find the Particular Integral of. $(D^2+1)y=\sin x$. | 2 | K1 | CO4 |
| 8. | Reduce the equation $(x^2D^2+xD+1)y=\log x$ into an equation with constant coefficients. | 2 | K2 | CO4 |
| 9. | Find the Laplace transform of $e^{-t}\sin 2t$. | 2 | K1 | CO5 |
| 10. | State initial value theorems on Laplace transforms. | 2 | K1 | CO5 |

## PART – B

(5 x 16 = 80 Marks)

| Q.No. | | | Questions | Marks | KL | CO |
|---|---|---|---|---|---|---|
| 11. | a) | i. | If f(z) is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\lvert f(z)\rvert = 0$. | 8 | K5 | CO1 |
| | | ii. | Prove that $u = e^x(x\,Cos\,y - y\,Sin\,y)$ is harmonic and hence find the analytic function $f(z) = u + iv$. | 8 | K5 | |
| | | | (OR) | | | |
| | b) | i. | Determine the bilinear transformation which maps the points $w = i, 1, -i$ on to $z = 0, 1, \infty$. | 8 | K5 | CO1 |
| | | ii. | Determine the image of the infinite strips $\frac{1}{4} < y < \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. | 8 | K5 | |
| 12. | a) | i. | Using Cauchy's integral formula, evaluate $\int_c \left(\frac{4-3z}{z(z-1)(z-2)}\right) dz$ where c is $\lvert z\rvert = \frac{3}{2}$. | 8 | K5 | CO2 |
| | | ii. | Construct the Laurent's series expansion of $f(z) = \frac{7z-2}{z(z+1)(z+2)}$ in $1 < \lvert z+1\rvert < 3$. | 8 | K3 | |
| | | | (OR) | | | |
| | b) | i. | Evaluate $\int_C \frac{(z-1)dz}{(z-1)^2(z-2)}$, where C is the circle $\lvert z-i\rvert = 2$ using Cauchy's residue theorem. | 8 | K5 | CO2 |
| | | ii. | Make use of contour integration to evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$ | 8 | K3 | |
| 13. | a) | | Verify Green's theorem in the XY-plane for $\int_c (3x^2-8y^2)dx + (4y-6xy)dy$, where C is the boundary of the region defined by $x=y^2$, $y=x^2$. | 16 | K5 | CO3 |
| | | | (OR) | | | |
| | b) | | Verify Gauss divergence theorem for $\vec{F} = 4xz\,\vec{i} - y^2\,\vec{j} + yz\,\vec{k}$ over the cube bounded by x=0,x=1,y=0,y=1,z=0,z=1. | 16 | K5 | CO3 |
| 14. | a) | i. | Solve $(D^2-4D+4)y = e^{2x} + \cos 4x$. | 8 | K3 | CO4 |
| | | ii. | Solve $(x^2D^2 - xD + 1)y = \sin(\log x)$. | 8 | K3 | |

(OR)

| | | | | | |
|---|---|---|---|---|---|
| b) | i. | Solve $(D^2+a^2)y=\tan ax$ by method of variation of parameters. | 8 | K3 | CO4 |
| | ii. | Solve $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos[\log(1+x)]$. | 8 | K3 | |
| 15. a) | i. | Find the Laplace transform of the following triangular wave function given by $f(t)=\begin{cases} t & ,0<t<a \\ 2a-t & ,\pi<t<2a \end{cases}$ With $f(t+2a)=f(t)$. | 8 | K3 | CO5 |
| | ii. | Apply convolution theorem to evaluate $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$. | 8 | | |

(OR)

| | | | | |
|---|---|---|---|---|
| b) | Make use of Laplace transform to solve the differential equation $y''-3y'+2y=e^{2t}$ with $y(0)=-3, y'(0)=5$. | 16 | K3 | CO5 |